TD n° 1 de Mécanique
Calcul Vectoriel

Corrigés des exercices

Exercice 1 :

1-

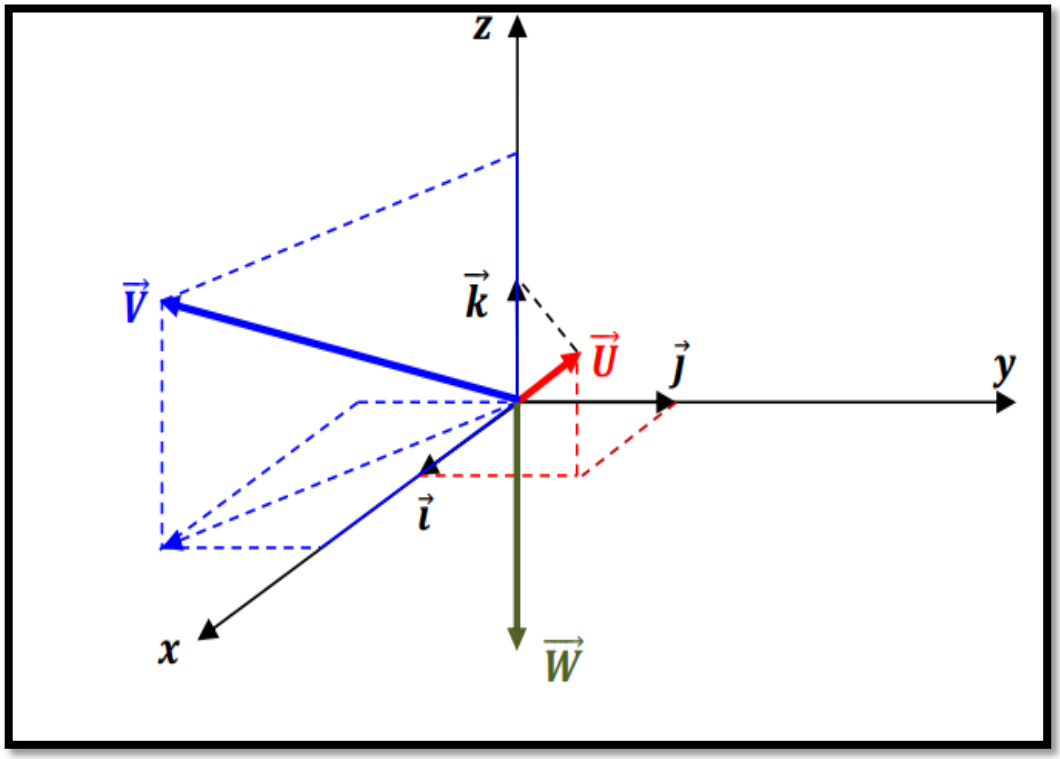
$\vec{U} + \vec{V} = 2\vec{\imath} + 4\vec{\jmath}$, $\vec{W} - \vec{X} = -5\vec{\imath} - \vec{\jmath}$, $-3\vec{Y} = -3\vec{\imath} + 9\vec{\jmath}$, $\vec{U} + 2\vec{V} + 3\vec{W} = -4\vec{\imath} + 10\vec{\jmath}$

2- Les composantes du vecteur $\vec{N}$: $\vec{N} \begin{pmatrix} 2 \\ 4 \end{pmatrix}$

3- Module : $\|\vec{N}\| = \sqrt{(2)^2 + (4)^2} = 2\sqrt{5}$.

Exercice 2 :

1-



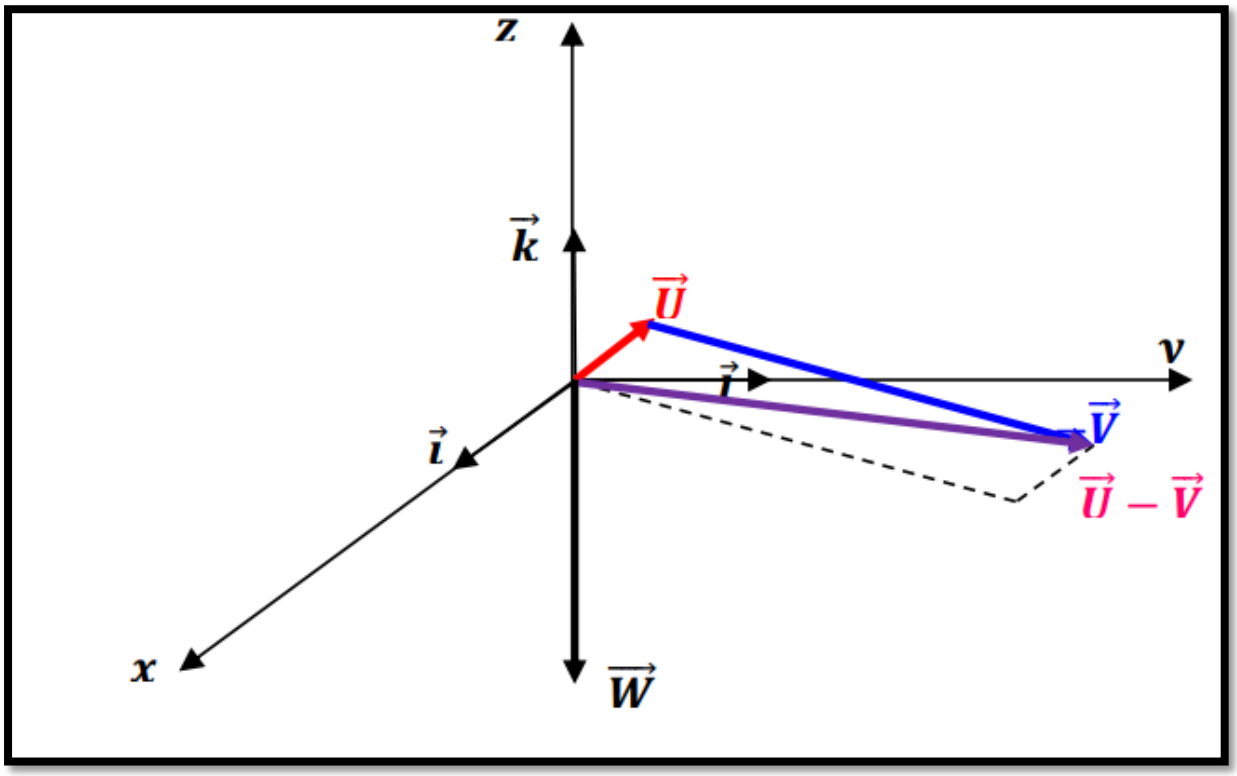
2- $\|\vec{U}\| = \sqrt{(1)^2 + (1)^2 + (1)^2} = \sqrt{3} = 1.73$

$\|\vec{V}\| = \sqrt{(2)^2 + (-1)^2 + (2)^2} = \sqrt{9} = 3$

$\|\vec{W}\| = \sqrt{(0)^2 + (0)^2 + (-2)^2} = \sqrt{4} = 2$

3- $\vec{U} = \|\vec{U}\| \times \vec{u} \Rightarrow \quad \vec{u} = \frac{\vec{U}}{\|\vec{U}\|} = \frac{1}{\sqrt{3}}(\vec{\imath} + \vec{\jmath} + \vec{k})$.

4-



$\vec{U} - \vec{V} = (\vec{\imath} + \vec{\jmath} + \vec{k}) - (2\vec{\imath} - \vec{\jmath} + 2\vec{k}) = -\vec{\imath} + 2\vec{\jmath} - \vec{k}$

$\|\vec{U} - \vec{V}\| = \sqrt{(-1)^2 + (2)^2 + (-1)^2} = \sqrt{6}$

5-

a- $\vec{U}.\vec{V} = (\vec{\imath} + \vec{\jmath} + \vec{k}) . (2\vec{\imath} - \vec{\jmath} + 2\vec{k}) = 2\vec{\imath}^2 - \vec{\jmath}^2 + 2\vec{k}^2 = 2 - 1 + 2 = 3$

b- $\vec{U} \wedge \vec{V} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ 1 & 1 & 1 \\ 2 & -1 & 2 \end{vmatrix} = \vec{\imath} \begin{vmatrix} 1 & 1 \\ -1 & 2 \end{vmatrix} - \vec{\jmath} \begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix} + \vec{k} \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix}$

$= \vec{\imath}(1 \times 2 - (-1) \times 1) - \vec{\jmath}(1 \times 2 - 1 \times 2) + \vec{k}(1 \times (-1) - 1 \times 2)$

$= 3\vec{\imath} + 0\vec{\jmath} - 3\vec{k}$

c- $(\vec{U} \wedge \vec{V}) \wedge \vec{W} = (3\vec{\imath} - 3\vec{k}) \wedge (-2\vec{k}) = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ 3 & 0 & -3 \\ 0 & 0 & -2 \end{vmatrix}$

$$=\vec{\imath}\begin{vmatrix}0 & -3\\ 0 & -2\end{vmatrix}-\vec{\jmath}\begin{vmatrix}3 & -3\\ 0 & -2\end{vmatrix}+\vec{k}\begin{vmatrix}3 & 0\\ 0 & 0\end{vmatrix}= 0\vec{\imath}+6\vec{\jmath}+0\vec{k} = 6\vec{\jmath}$$

d- $(\vec{V}\wedge\vec{W}).\vec{U}$

$$(\vec{V}\wedge\vec{W}) = \begin{vmatrix}\vec{\imath} & \vec{\jmath} & \vec{k}\\ 2 & -1 & 2\\ 0 & 0 & -2\end{vmatrix} = \vec{\imath}\begin{vmatrix}-1 & 2\\ 0 & -2\end{vmatrix}-\vec{\jmath}\begin{vmatrix}2 & 2\\ 0 & -2\end{vmatrix}+\vec{k}\begin{vmatrix}2 & -1\\ 0 & 0\end{vmatrix}$$

$$= 2\vec{\imath}+4\vec{\jmath}+0\vec{k}$$

$$(\vec{V}\wedge\vec{W}).\vec{U}= (2\vec{\imath}+4\vec{\jmath}+0\vec{k}).(\vec{\imath}+\vec{\jmath}+\vec{k}) = 2\vec{\imath}^2+4\vec{\jmath}^2+0\vec{k}^2 = 6$$

6- $\vec{U}.\vec{V}= \|\vec{U}\|\|\vec{V}\|\cos\widehat{U.V}$

$$\Rightarrow \cos\widehat{U.V}= \frac{\vec{U}.\vec{V}}{\|\vec{U}\|\|\vec{V}\|}= \frac{3}{1.73\times 3} = 0.578$$

$$\vec{U}.\vec{V}= 54^0 68'$$