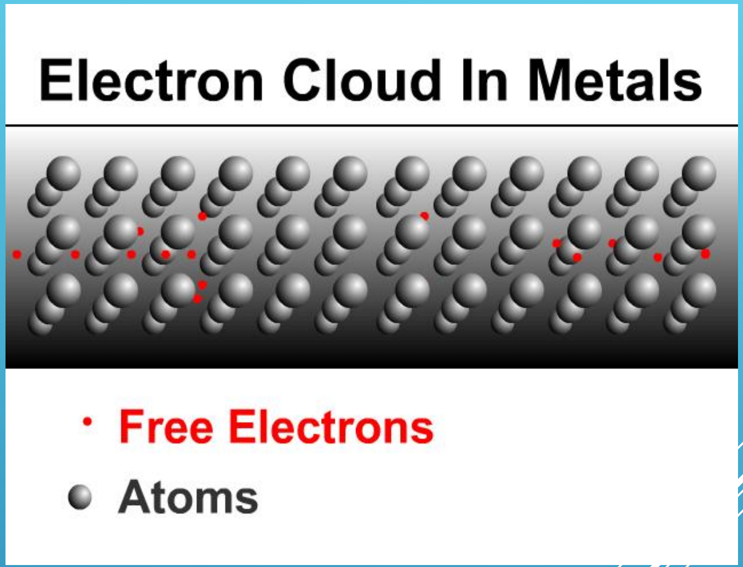
Electric conductors and capacitors

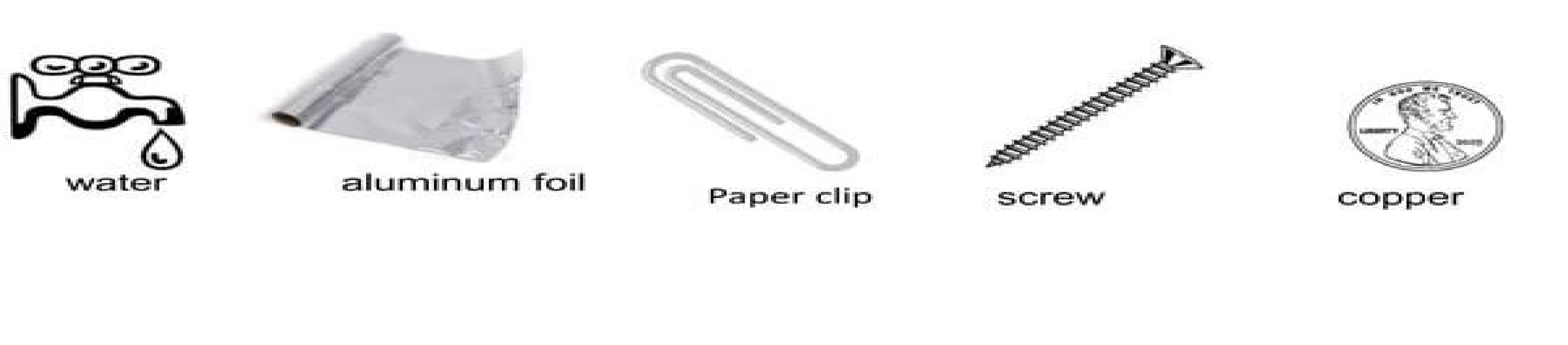
Difinition

- **Electrical conductors** are materials through which electrons move freely.
- When such a material is placed in an electric field, **the free electrons move** in a direction opposite to the field.

$$E = \frac{\sigma}{\varepsilon_0}$$



Example:

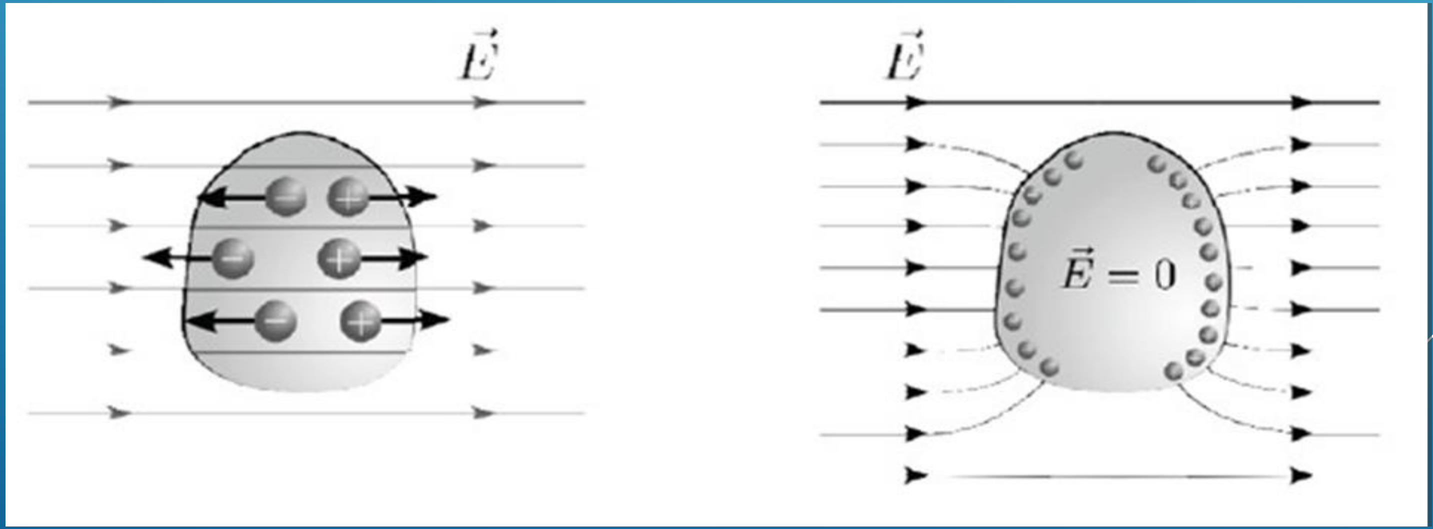


Conductor In Electrostatic Equilibrium

- A conductor is said to be in if all charges are immobile (no charge displacement in this medium) electrostatic equilibrium .

- Charges in <u>a neutral </u>conductor placed in a uniform electrostatic field move to cancel out the field in the conductor. This changes the external fiel

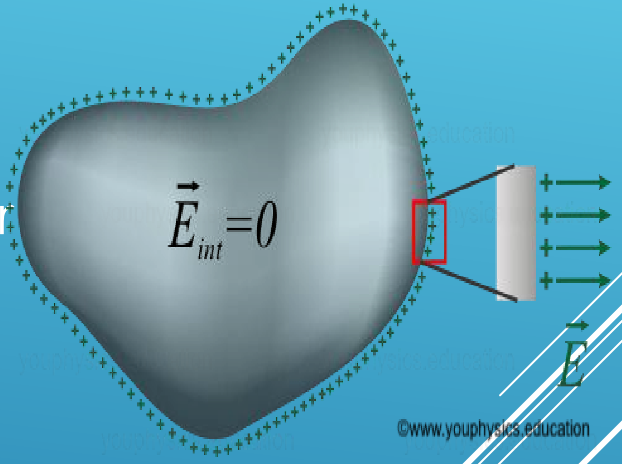
Out of equilibrium **In equilibrium**



Properties of a conductor in equilibrium

A- The Electric Field

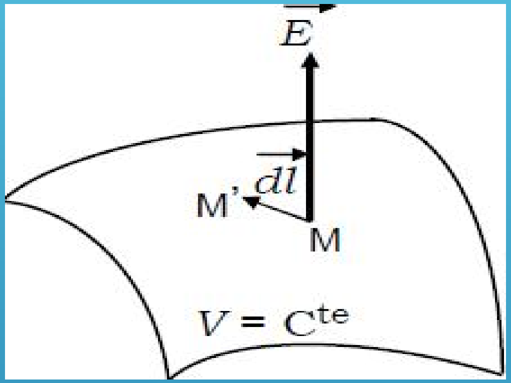
- electrostatic equilibrium means that the **charges inside a conductor are immobile**, and therefore not subject to any force. this condition translates **into a zero** electrostatic force for each charge. $\vec{F} = q\,\vec{E} = \vec{0}$ therefore the electric field $\vec{E} = \vec{0}$



- for the same reason, **the field on the surface** of the conductor must be **perpendicular** to this surface, because if there were a parallel component, the free charges would migrate on the surface of the conductor

B- The Conductor In Equilibrium Constitutes An Equipotential Volume

- Indeed, the potential difference (ddp) between any two points M and M' is defined by $dV = -\vec{E}.\overrightarrow{MM'}$ or E=0 for a conductor in equilibrium ⇒ **V=cte.**



- As the potential is the same at all points of the conductor, the external surface is an **equipotential surface**. We find that the field is normal to this surface.

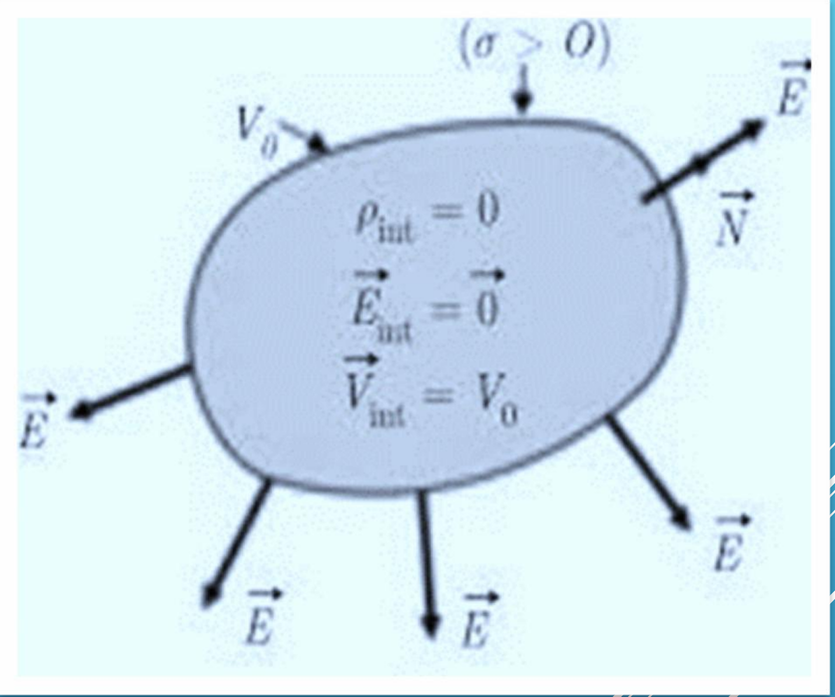
C. Distribution Of Charges

We have $\varnothing = \oint \vec{E} \cdot \vec{ds} = \sum Q_{int} / \varepsilon_0$ (GT) and

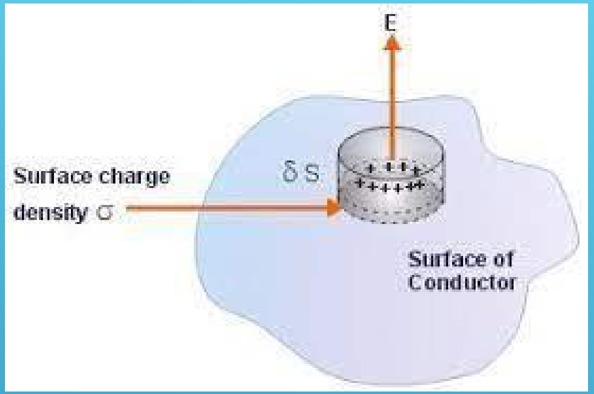
$\vec{E}_{int} = \vec{0}$ so Qint=0

$\boxed{\rho = 0}$

The total electric charge inside the conductor is then said to be zero, and the charge is localized on the surface of the conductor (**surface distribution** σ).



The Electric field in the immediate vicinity of a conductor and the surface electric charge (coulomb's theorem)

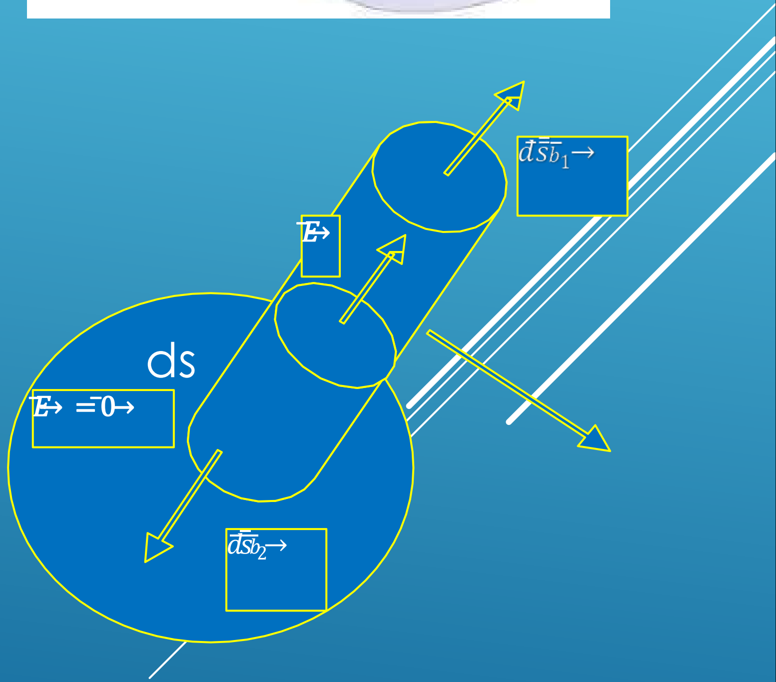


The electric flow is made up of three terms:

- Flux through **the lateral** surface (zero) ($\vec{E} \perp \vec{dS}$)
- Flux through the interior base (zero) (E=0)

Only the flow remains through the **exterior base**

$d\varnothing = E.dS$



Furthermore, if σ is the surface charge density, the charge contained in the cylinder is: **dq =** $\sigma.dS$
By applying Gauss' theorem:

$E.dS = (\sigma.dS)/\varepsilon_0 \Rightarrow \mathbf{E = \sigma/\varepsilon_0}$

Electrostatic pressure

- Charges on the surface of a conductor are subject to repulsive forces from other charges.
- The force exerted per unit area, or electrostatic pressure, can be calculated by multiplying the average electric field on the surface of the conductor by the charge per unit area.

- The average electric field is according to the above:

$$E = \sigma / 2\varepsilon_0$$

- The electrostatic pressure is:

$$p = \sigma . E = \sigma^2/\varepsilon_0$$

Capacity Of A Conductor In Electrostatic Equilibrium

- On a conductor isolated in space, let us deposit a charge q: this results at every point in space a charge q'=aq, we will have at every point in space, a field $(E')^{\rightarrow} = \alpha \; E^{\rightarrow}$
- since E and q are proportional and this is true in particular on the conductor whose potential is V.
- We write this in the form:

Q= C.V

- The proportionality constant C is called the self-capacitance of the isolated conductor:

C= $Q/V = coul oumb/volt$ **= farad**

The farad is a very large unit, we use sub multiples:

microfarad: 10^{-6} F (µF)

nanofarad: 10^{-9}F (nF)

picofarad: 10^{-12} F (pF)

Potential Energy Of A Conductor In Electrostatic Equilibrium

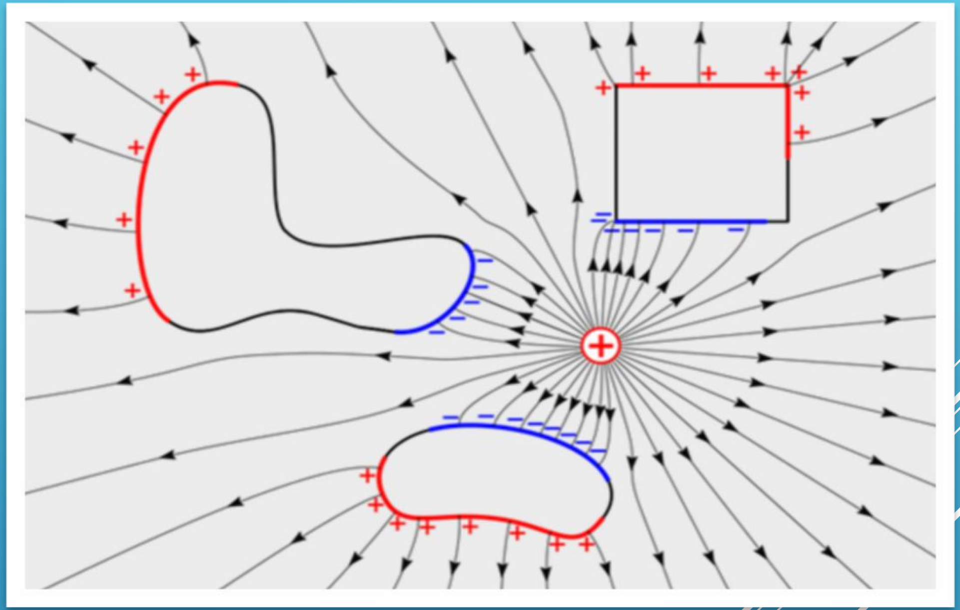
A conductor in electrostatic equilibrium carrying charge Q, let V be its potential and C its capacity, its potential energy is written as:

$$E_p = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}C\,v^2 = \frac{1}{2}Q\,v$$

(this energy is always positive)

Electrostatic Influence Between Conductors

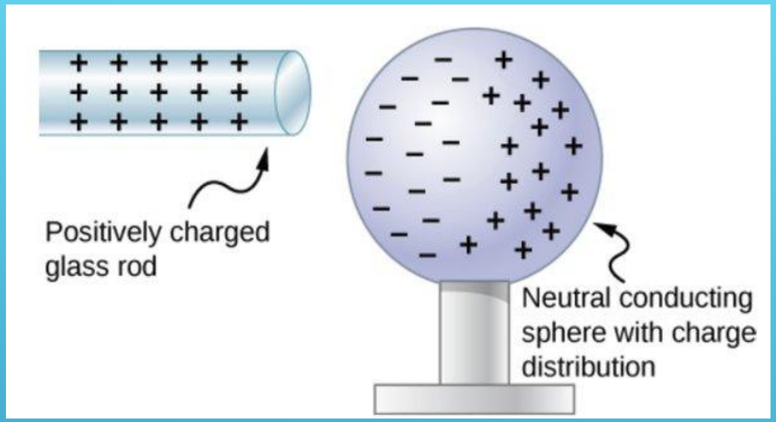
If we place a conductor in an electric field, the positive charges go in the same direction as the field and the negative charges go in the opposite direction, creating two poles, one positive and one negative.



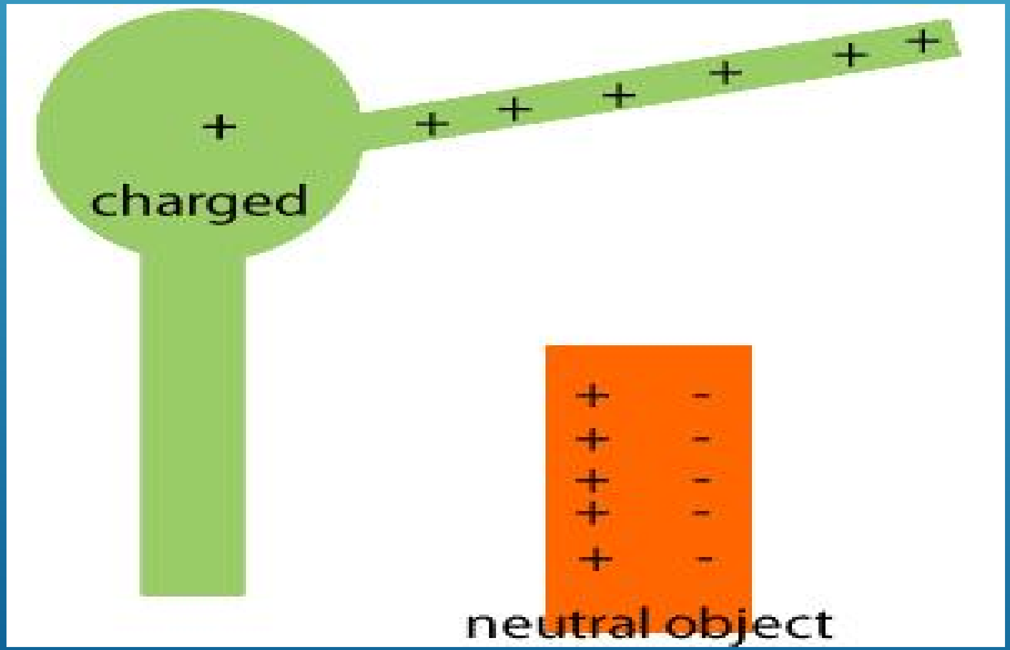
There are **two types** of electrostatic **influence** in the presence of two charged conductors.

1- Partial influence

Let us consider an electrically neutral conductor A. Let us approach the latter, a positively charged conductor B, as shown in the figure. Conductor B creates an electric field in space and in particular in conductor A $\vec{E}$



(B) (A)



- The free electrons of conductor A will, under the action of this field, move in the opposite direction to $\vec{E}$. These electrons gradually accumulate on the face facing B and form negative charges in equilibrium, the result of which is –Q

- Conversely, positive charges, the result of which is +Q, will appear on the other face of conductor A, which result from electrification by influence

- The accumulation of charge(+) on one side and charge(-) on the other side, creates an electric field $(\vec{E}')$ inside the conductor A which opposes E inside the conductor A which opposes $\vec{E}$

- Inside conductor A, the free electrons only stop moving when the total electric field disappears. The system formed by the two conductors then reaches a state of polarised equilibrium.

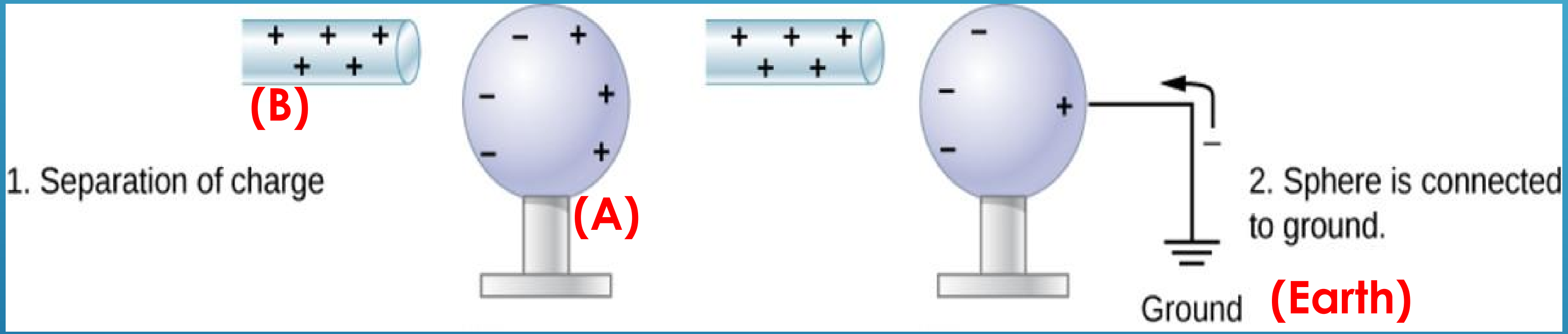
Corresponding elements theorem: Faraday's theorem

The charges carried by two corresponding surface elements facing each other are equal and of opposite sign.

- If conductor A is connected to earth, positive charges flow to earth, and the charge in conductor A will be Q'A is negative due to the partial influence of the earth.

Appearance of charges (−) on the part of A close to B .The (+) charges of A go into the **earth.**

V **A =** V **(earth) = 0 But** Q **A ≠ 0**



Phenomenon of influence does not modify the potential of the conductor, but modifies its total charge and the distribution of this charge.

2- Total Influence

There is total influence when the influencing conductor B surrounds the influencing conductor A

This is three case :

1-Case where B is neutral: $Q_B = 0$

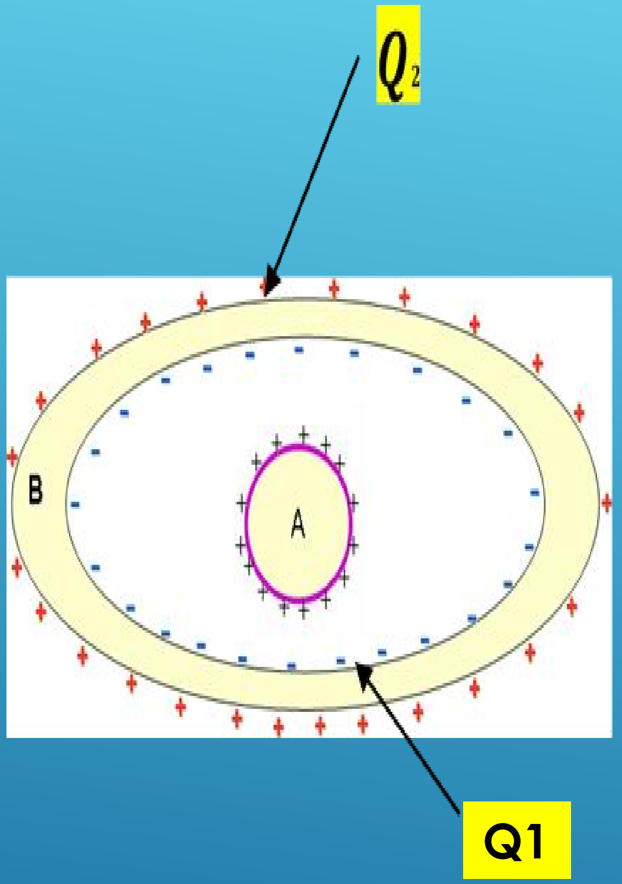
<u>We put A into B :</u>

For A : $Q_A > 0$

For B: $Q_B = Q_1 + Q_2$

Withe : $Q1= -QA$

$Q2= +QA$



So : $QB = (+Q_1) + (QA) = 0$

2- Case where B is hollow and : $QB \neq 0$

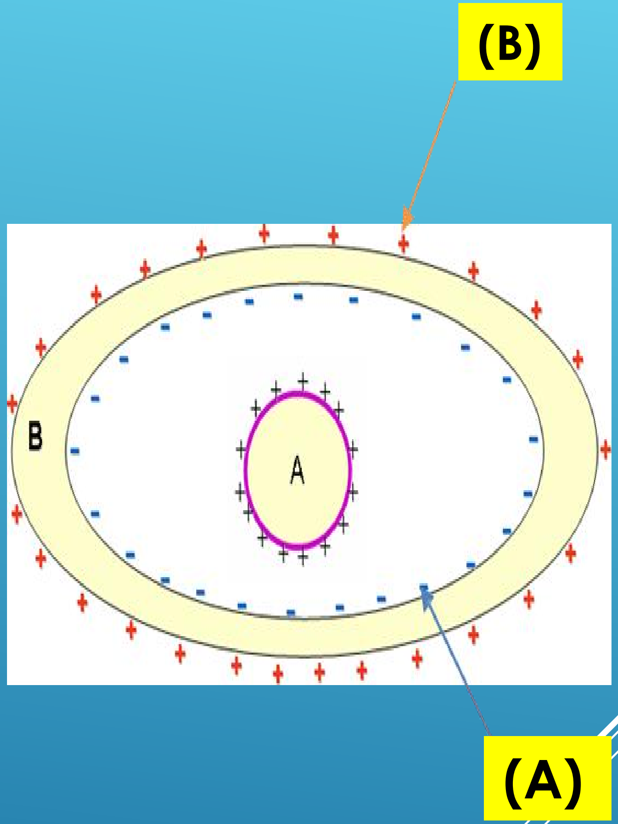
We put A in B :

For A : $\boldsymbol{Q}\ A > 0$

For B : $\boldsymbol{Q}\ B = \boldsymbol{Q}\ 1 + \boldsymbol{Q}\ 2$

Withe : $\boldsymbol{Q}\ 1 = -\boldsymbol{Q}\ A$

$\boldsymbol{Q}\ 2 = \boldsymbol{Q}\ B - \boldsymbol{Q}\ 1$



So: $\boxed{Q2 = \boldsymbol{Q}B + \boldsymbol{Q}A}$

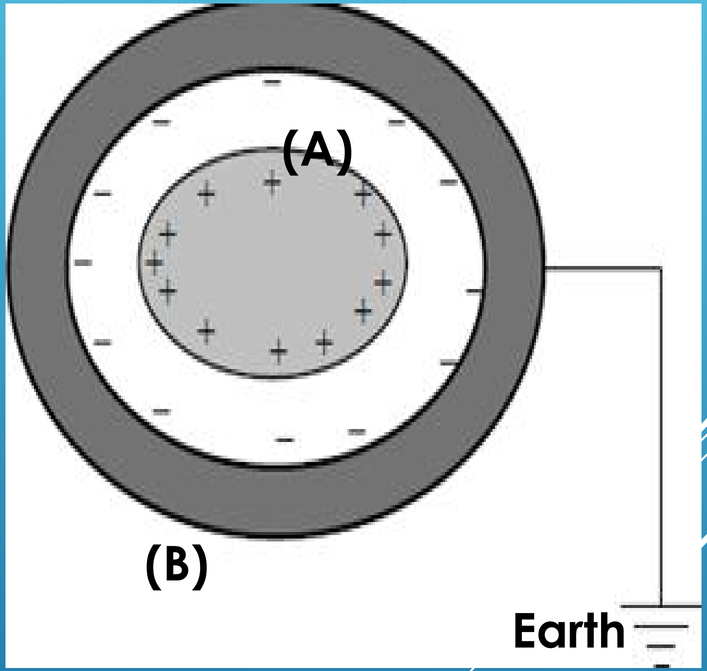
3- Case B connected to the earth

No charge on its external side

we connect B to the earth

Q1=0

So : $\boxed{V_B = V_{earth}} = 0$



Capacitors

Definition

A capacitor is a system of two conductors that carries equal and opposite charges, separated either by a air or by an isolator
We define the capacitance of capacitor by :

C= $Q\ /V$ Unit: Farad(F)

Q: charge in one plate
V: potential between the two plates

Calculating Capacitance

- A general method for calculating the capacitance of a capacitor consists of calculating the relationship between its charge Q and the voltage applied between the two plates (V 1 - V 2).
- Deduce the field between the armatures then calculate V 1 - V 2 using the expression for the circulation of the electric field:

$$V_1 - V_2 = \int_1^2 E.dl = \frac{Q}{V}$$

This method only applies, of course, in simple geometric cases, such as those we will examine now.

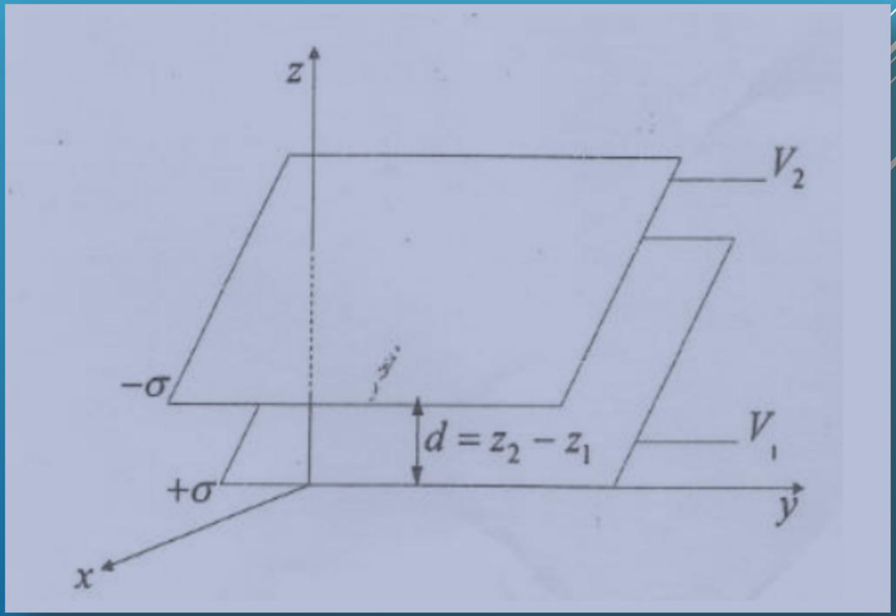
Calculating Capacitance

1-Parallel-plate capacitor

- The plane capacitor is made up of two parallel conducting planes spaced d apart and with surface S.

- Let the load Q be distributed regularly on each reinforcement with the (uniform) surface density:

$$Q = \sigma \; / S$$



- ✓ The electrostatic field between the armatures is the composition of the fields resulting from the two infinite planes

$$\vec{E} = \overrightarrow{E_1} + \overrightarrow{E_2} = \frac{\sigma}{2\varepsilon_0}\vec{k} + - \frac{\sigma}{2\varepsilon_0}(-\vec{k}) \;\longrightarrow\; \vec{E} = \frac{\sigma}{\varepsilon_0}\vec{k}$$

$$V_1 - V_2 = \mathrm{V} = \int E.dz = \int_{z_2}^{z_1} \frac{\sigma}{\varepsilon_0}(z_2 - z_1) \;\longrightarrow\; \mathrm{V} = \frac{\sigma}{\varepsilon_0}\mathrm{d}$$

- ✓ The capacitance of a parallel plane capacitor is

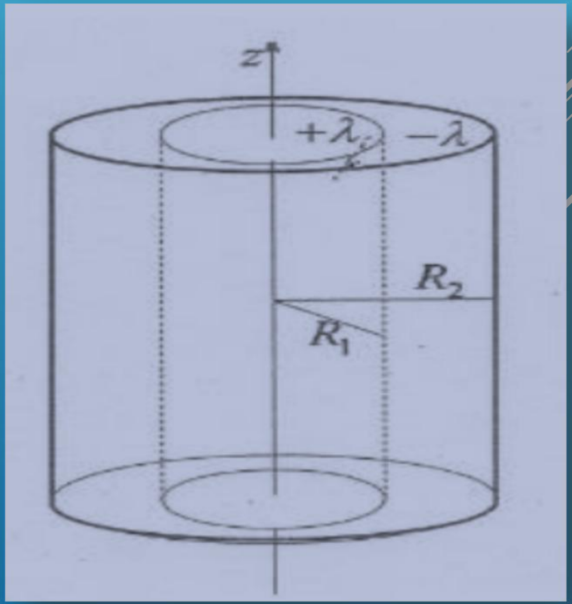
$$\mathrm{C} = \frac{Q}{V} = \varepsilon_0 \frac{S}{d}$$

Cylindrical Capacitor

- We consider two unlimited and coaxial cylinders of radius R 1 and R 2 with R 1< R 2

- We are looking for the capacity of a section of height h. We denote by Q the load carried by the internal reinforcement over the length h

- The calculation of E (which is radial) at a distance r (R1< r<R2) from the axis, is done by the immediate application of Gauss' theorem to a cylinder of radius r closed at these two ends:



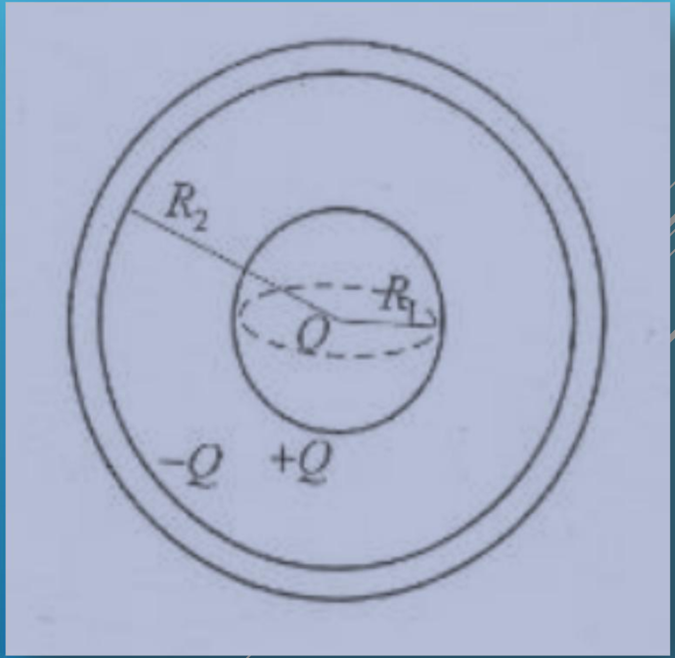
- $E \cdot 2\pi r h = Q/\varepsilon_0$
- $V_1 - V_2 = V = \int_{R_1}^{R_2} E.dr = \frac{Q}{2\pi\varepsilon_0 h} \int_{R_1}^{R_2} \frac{1}{r} dr$

$$\rightarrow V = \frac{Q}{2\pi\varepsilon_0 h} \ln(R_2/R_1)$$

$$C = \frac{2\pi\varepsilon_0 h}{\ln(R_2/R_1)}$$

Spherical capacitor

- Consider the capacitor formed by two thin concentric spheres, of radius R_1 and R_2 with $R_1 < R_2$
- By applying Gauss' theorem, the calculation of the field E produced by a sphere at a distance r ($R_1 < r < R_2$) is known
- we write :



$$E(r) = \frac{Q}{4\pi\varepsilon_0}\frac{1}{r^2}$$

$$V_1 - V_2 = V = \int_{R_1}^{R_2} E.dr = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$C = \frac{Q}{V} \rightarrow C = 4\pi\varepsilon_0 . \frac{R_1 R_2}{R_2 - R_1}$$

Energy stored in a capacitor

- the energy stored by a capacitor, loaded with charge Q, is proportional to the square of the voltage applied between its plates.
  Its expression is:

$$W = \frac{1}{2} C V^2$$

Knowing that **Q = C.V** we can write

$$W = \frac{1}{2} \frac{Q^2}{C}$$


Thank you