Electricity course

Table des matières

Objectifs

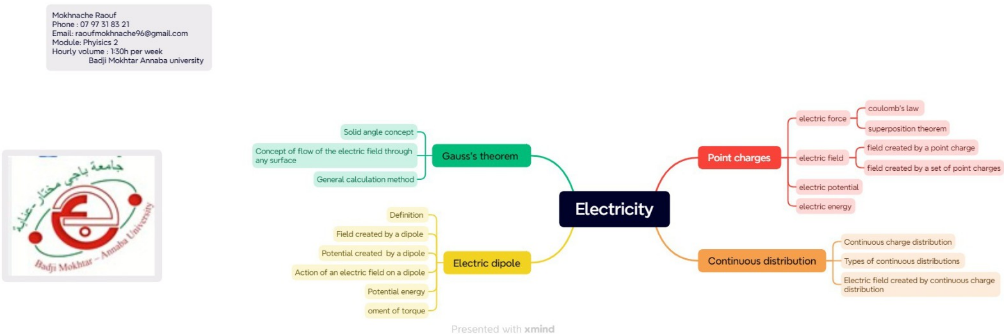
1-Understand the Fundamentals of Electrostatics
2-Apply Mathematical Tools to Physical Problems
3-Explore the Concept of Electric Dipoles
4-Master Gauss's Theorem and Its Applications
5-Develop Problem-Solving Skills

Introduction

Welcome to this course on electrostatics, where you'll explore the behavior of electric charges at rest and their interactions. We begin with **Point Charges**, examining the forces and electric fields they create. Next, we'll cover **Continuous Charge Distributions**, learning to calculate electric fields for different geometries.

We'll then study the **Electric Dipole**, understanding its properties and how it interacts with external fields. Finally, we'll master **Gauss's Theorem**, a key tool for simplifying electric field calculations in symmetric systems.

This course will equip you with a solid understanding of electrostatics, preparing you for advanced physics and engineering challenges.



Pre-requises

1-Basic Mathematics:

2-Familiarity with Vector Analysis

II Chapter 1 : Point charges

1. Introduction

What is the difference between static electricity and current?

In **Static Electricity (Electrostatic)** , charges build up on an object but do not flow. Electrons move from one surface to another, creating an imbalance.

Electric Current involves the continuous flow of electric charge. Electrons move through a conductor, creating a flow of current.

2. Electrification Experiments

Electrification (**charging**) represents a phenomenon of charge transfer. There are three types of electrification of an object by:

- Friction
- Conduction
- Influence

2.1. Electrification by Friction

We rub a plastic or glass ruler with wool and bring it close to the small pieces of paper, it attracts it. Without friction nothing happens, after friction the ruler will be electrified (charged). We say that there is a mechanical removal of electrons from a body

2.2. Electrification by Conduction

Contact electrification occurs on a neutral object when a charged object is in **contact** with it.

During conduction the same charge is created in a neutral object. Electrons will transfer from a negative object to a neutral object making it negative.

Charging By Conduction Facts :

- **Contact**
- **Same** charge
- **Permanent** (with electron transfer)

2.3. ELECTRIFICATION BY INDUCTION (INFLUENCE)

You can induce a charge in a neutral object by moving a charged object close to it.

Induction electrification or influence creates a temporary and opposite charge in that other object with **no contact.**

This is considered temporary because no electrons are transferred and neutrality returns when the close charged object is removed.

Charging By Induction Facts

- **No contact**
- **Opposite charge**

- **Temporary (no electron transfer)**

3. Electric charge

Définition **:**

Electric charge it is a scalar quantity, representing a fundamental property of matter that helps explain certain phenomena (electrostatics, electromagnetism, etc.).

There are two types of electric charge, **positive** and **negative**.

Elementary charge: This is the smallest amount of charge Q= |e| = **1.602176634x10-19 C**

the electrical charge of an:

electron : Qe = -e = -1.602176634x10-19 C

and **proton** : Qp = +e = +1.602176634x10-19 C

Two charges of the **same** sign **repel** each other, and two charges of **opposite** signs **attract** each other.

Définition **: Point charge**

it is an electric charge localized at a dimensionless point.

Définition **: conservation of electric charge**

In an isolated body, the algebraic sum of electric charges remains constant: qfinal = qinitial

4. Conductive and insulating materials

Two materials are distinguished: conductors and insulators.

Conductive materials : In conductors, electric charges are free to move and are distributed throughout the material. An electrical conductor therefore carries the electric current (iron, aluminum, salt water, etc.).

Insulating materials (dielectrics) : An electrical insulator is a medium that does not conduct electric current, as it does not allow the passage of free electrons from one atom to another (ebonite, glass, porcelain, plastics, etc.). The insulator becomes charged by friction.

5. Coulombs law

Interaction between two point charges q1 and q2:

Two charges q1 and q2 spaced r apart placed in a vacuum. The first exerts on the second a force F12, the second exerts on the first a force F21. Coulomb's law can be used to determine the electrostatic force, which is, written as :

$$\vec{F}_e = \vec{F}_{12} = -\vec{F}_{21} = K\frac{q_1 q_2}{r^2}\,\vec{u} \Rightarrow F_e = F_{12} = F_{21} = K\frac{|q_1|.|q_2|}{r^2}$$

Couloumb law

where k is a constant and equals to 1/4 π ε0. Here, ε0 is the epsilon naught and it signifies permittivity of a vacuum. The value of k =9 × 109 Nm2/ C2 when we take the S.I unit of value of ε0 is 8.854 × 10-12 C2 N-1 m-2.

where k is a constant and equals to $1/4\,\pi\,\varepsilon_0$. Here, ε_0 is the epsilon naught and it signifies permittivity of a vacuum. The value of $k = 9 \times 10^9\ Nm^2/C^2$ when we take the S.I unit of value of ε_0 is $8.854 \times 10^{-12}\ C^2\ N^{-1}\ m^{-2}$.

If the two charges have the same sign then the force is **repulsion force**

If the two charges have different signs, then the force is **attraction force**

✎ ***Remarque*** :

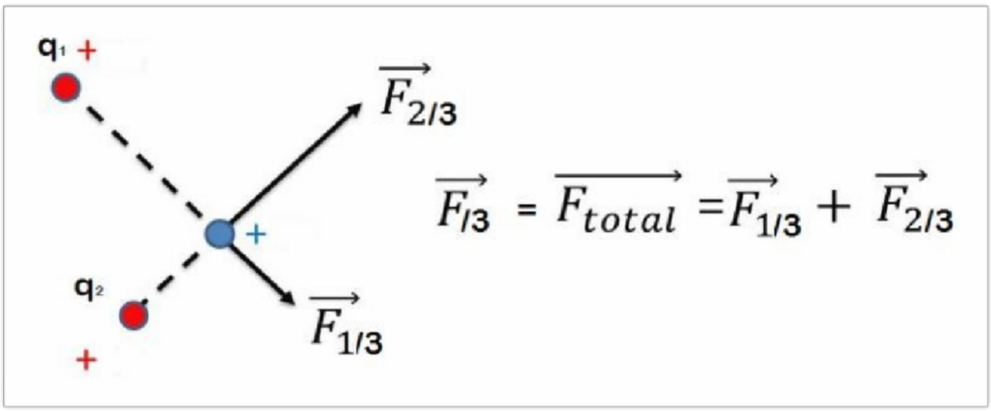
In a medium other than vacuum, ε0 will be replaced by ε = ε0εr where εr represents the relative permittivity, so the force is given by the following relationship:

$$\vec{F}_e = \frac{q_1 q_2}{4\pi\varepsilon_0\varepsilon_r r^2}\,\vec{u} \Rightarrow F_e = \frac{|q_1|.|q_2|}{4\pi\varepsilon_0\varepsilon_r r^2}$$

6. Superposition of Coulomb law

Assuming that there exist n immobile electric charges in a vacuum. Electrostatic force exerted by the n charges on a charge q located at a point **M** is :

The **superposition principle** allows us to determine the **total force** on a given charge due to any number of point charges acting on it.



Superposition Theorem for forces

7. The electrostatic field

Electric Fields

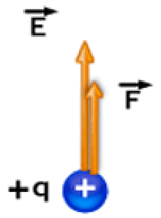
A static charge on an object will create an electric field. Learn to draw electric fields around single and multiple charges and solve for their value.

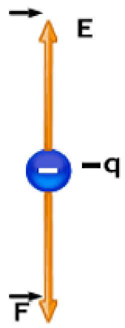
The basics of the electric field

- Electric field (E): area of electrical influence around a charged object.
- Variable (E)
- Unit: newton per coulomb (N/F)

$$E = \frac{F}{q}$$

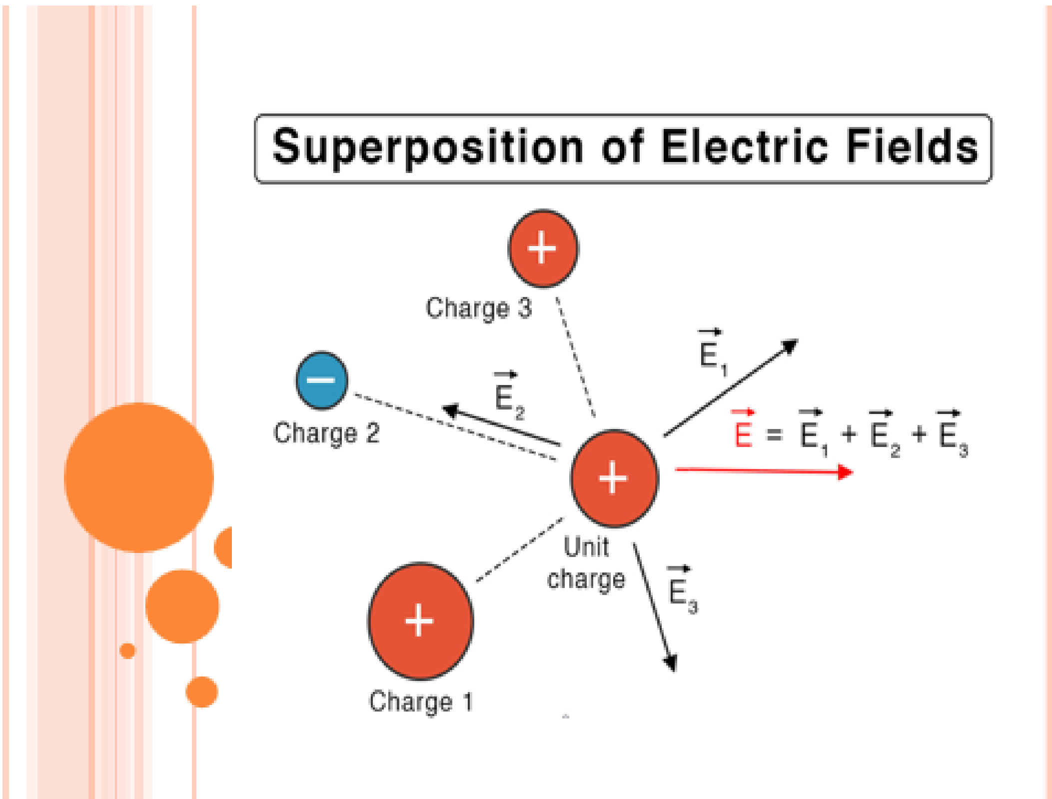
$$E = \frac{kq}{d^2}$$





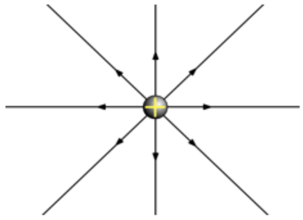
Superposition of an electric field

We use the same principale as with forces



8. Field lines

The lines for a positive charge point away from the charge



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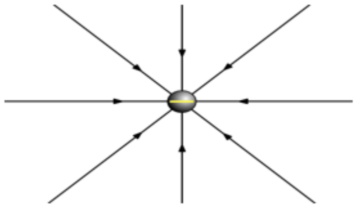
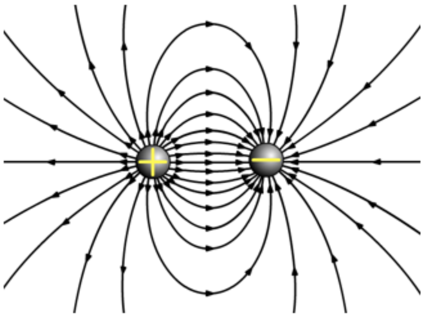
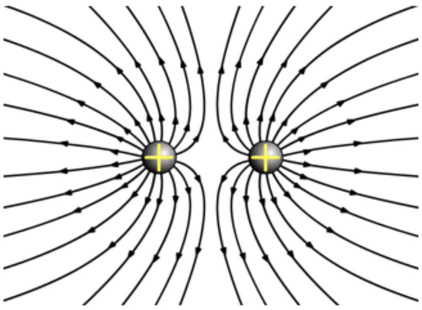




Case of two opposite charges (dipole)



Case of two positive charges

9. Units of charge

MILLI COULOMB	mC	$\times 10^{-3}$
MICRO COULOMB	μC	$\times 10^{-6}$
NANO COULOMB	nC	$\times 10^{-9}$
PICO COULOMB	pC	$\times 10^{-12}$

10. Electric Potential :

We saw in work and energy that Potential energy refer to the energy stored in a body that can be converted to a kinematic energy Here we apply the same context in electricity

So the Electric potential the electrical potential , the energy neede to move a point charge tought a distance r and it is calculated by

$$V = \frac{W}{q} = k\,\frac{q}{r} \qquad W = -q \cdot E \cdot r$$

W: Work done to move the charge

q: the electrical charge

k: coulomb's constant

Superposition principal

The same principal apply as with force, and electrical field the potential in a field is the equal to the sum of all electrical potentials of all charges

V = V1 + V2 + V3 + ...etc

11. Interaction Energy between two points charges :

The Interaction Energy between two points charges can be calculated using coulomb's law equation

$$Ep = K\,\frac{q_1 q_2}{r}$$

12. Relationship between Electrical field and Electrostatic Potential:

The relationship between electrical field and electrostatic potential can be expressed by the following

$$\vec{E} = -\overrightarrow{grad}(V) \qquad -dV = -\vec{E}.\,\vec{dl}$$

III Chapter 2 : Continuous distribution

1. Introduction

The electric field due to a small number of charged particles can be Easily calculated using the principle of layering.

But what if we have a very large number of charges spread across a region of space?

2. Continuous charge distribution

A continuous charges distribution is used to describe the charge of a macroscopic object.

While the electric charge is an integer multiple of the unit of electric charge, we can consider to be continuous.

Sometimes when we have a great number of point charges we treat them as if their number is infinite and **continuously distributed in the field**

In this case we talk **about continuous distribution**

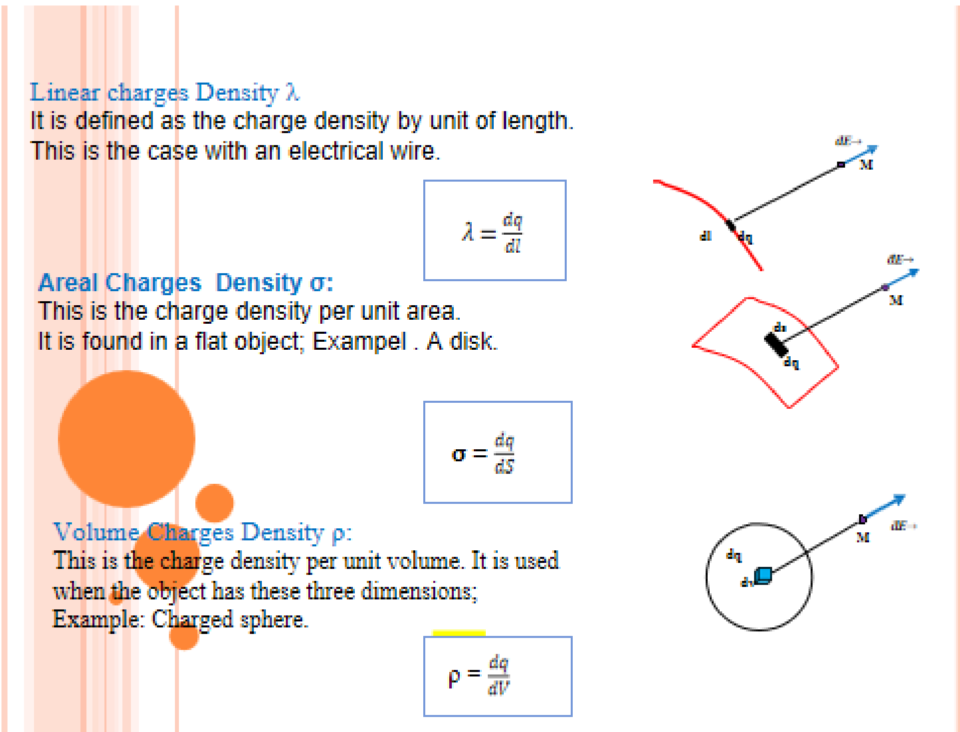
3. Types of continuous distribution

Three types of charge density can be defined, depending on the shape and dimensions of the object that creates the electric field:

Linear distribution: if the charges are distributed along a line

Surface distribution: if the charges are distributed along a surface

Volume distribution: if the charges are distributed along a volume



4. Electric field created by continuous charge distribution

If the loads are spread in a continuous distribution (per unit of the volume dq = ρ.dV, per unit of area dq = σ.dS ou per unit of length dq = λ.dl),

the summation becomes an integral, and the expression of the field becomes:

1. Volume charge density, $\rho = \frac{dq}{dv}$
2. Surface charge density, $\sigma = \frac{dq}{ds}$
3. Linear charge density, $\lambda = \frac{dq}{dL}$
4. Force exerted on a charge q_0 due to a continuous charge distribution,
$$\vec{F} = \frac{q_0}{4\pi\varepsilon_0}\int \frac{dq}{r^2}\,\hat{r}$$
5. Electric field due to a continuous charge distribution,
$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r^2}\,\hat{r}$$

Units Used

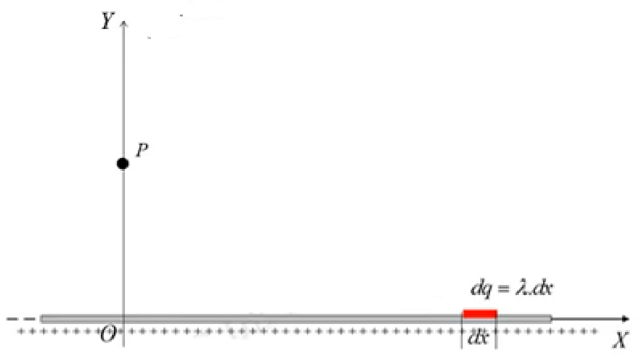
ρ is in Cm^{-3}, σ in Cm^{-2}, λ in Cm^{-1} and E in NC^{-1}.

5. Example of continuous linear distribution

The electrostatic field produced by a fine wire of infinite length et have a linear positive constant charge density λ

Let's calculate the electrostatic field produced by those charges on a point P

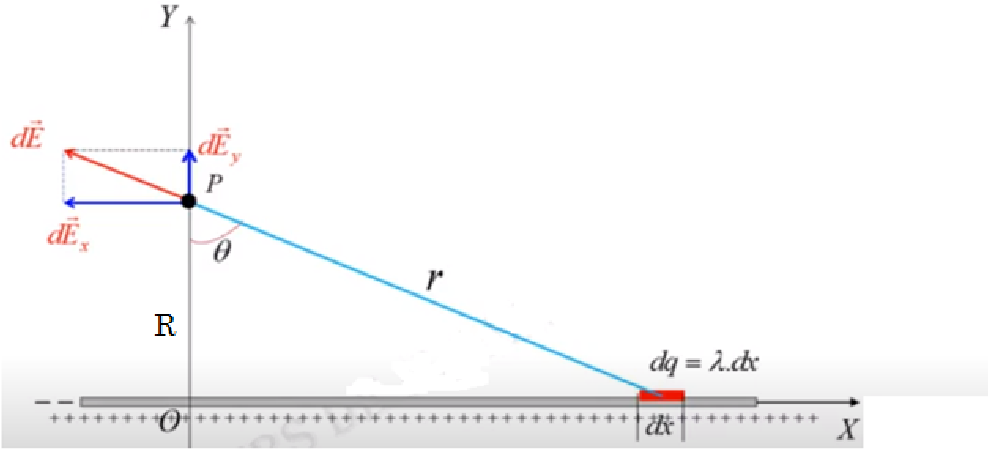
6. A



Here we take the rectilinear segment dx that has a charge dq = λ .dx

Here, the elementary field is located on the extension of the rectilinear segment of length r and connecting p to dq

7. B



8. C

Here when we project the field dE on the X and Y axis we find:

$\vec{dE} = \vec{dE_x} + \vec{dE_y}$

We have : $dE_x = dE.\cos\theta$ $\quad dE_y = dE.\sin\theta$

$dE = k\frac{dq}{r^2} = k.\frac{\lambda.dx}{r^2}$

So $E_x = k\int\frac{dx}{r^2}\cos\theta$ and $E_y = k\int\frac{dx}{r^2}\sin\theta$

Geometrically $x = R.\tan\theta \rightarrow dx = R.\frac{1}{\cos^2\theta}d\theta$

$r = \frac{R}{\cos\theta}$

9. D

$$E_x = \lambda.\text{k} \int \frac{dx}{r^2} \cos\theta = \lambda.\text{k} \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{(R/cos^2\theta)}{R/cos^2\theta} d\theta.\ \text{Sin}\theta$$

$$E_x = \frac{1}{R} \lambda.\text{k} \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} sin\,\theta\ d\theta = \frac{1}{R} \lambda.\text{k}\ [-cos\theta\ ]_{\frac{-\pi}{2}}^{\frac{\pi}{2}}$$

$$E_x = 0$$

$$E_y = \lambda.\text{k} \int \frac{dx}{r^2} \cos\theta = \lambda.\text{k} \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{(R/cos^2\theta)}{R/cos^2\theta} d\theta.\ \cos\theta$$

$$E_y = \frac{1}{R} \lambda.\text{k} \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} cos\,\theta\ d\theta = \frac{1}{R} \lambda.\text{k}\ [sin\theta\ ]_{\frac{-\pi}{2}}^{\frac{\pi}{2}}$$

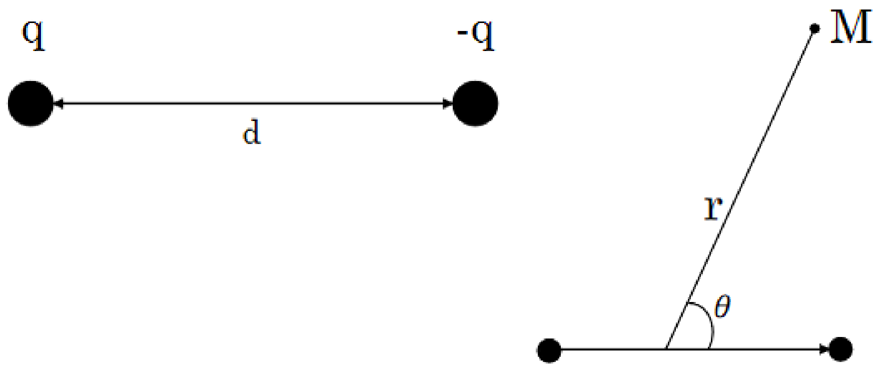
$$E_y = \frac{1}{R} \lambda.\text{k}\ [1 - \ - 1] = \frac{2}{R} \lambda.\text{k}\ = \frac{1}{2\pi\varepsilon_0} \frac{1}{R}$$

$$E_y = \frac{1}{2\pi\varepsilon_0} \frac{1}{R}$$

IV Chapter 3 : Electric dipole

1. Introduction

It’s a system created by two points charges with opposites signs and separated by a distance d



2. Electric potential of a dipole

The electric potential created by a dipole on a point M, spotted by it’s polar coordinates by is given by:

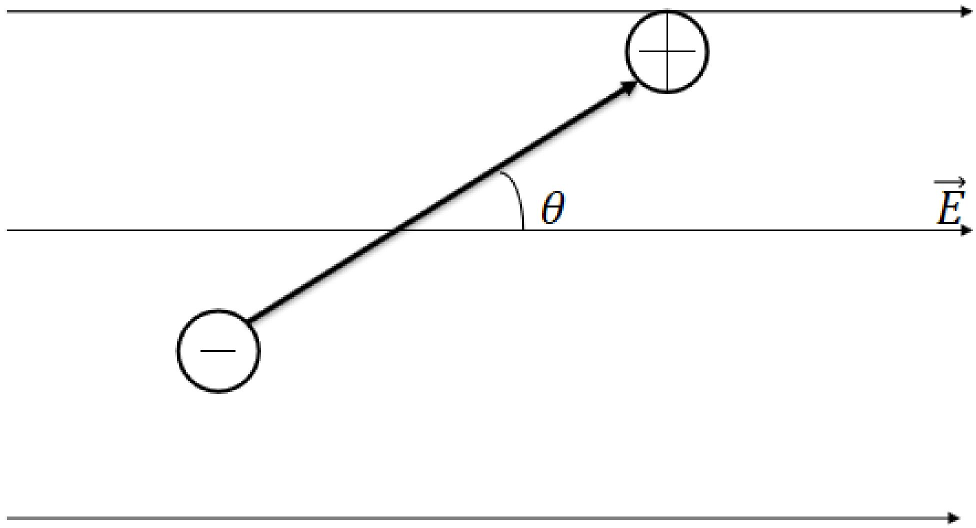
$$V = \frac{kd\cos\theta}{r^2}$$

3. Field created by a dipole

The electrical field created by this dipole is given by :

$$E = -\,\overrightarrow{grad}(V) \Rightarrow \begin{cases} E_r = -\dfrac{\partial y}{\partial r} = \dfrac{2kd\cos\theta}{r^3} \\ E_\theta = -\dfrac{1}{r}\dfrac{\partial y}{\partial \theta} = \dfrac{kd\cos\theta}{r^3} \end{cases}$$

4. Action of an electric field on a dipole



Potential energy

The potential energy (U) of an electric dipole in an external electric field is defined as the work done by the electric field to orient the dipole relative to its natural direction

$$Ep = - \vec{E}.\vec{p} = -E\ p\ \cos\theta$$

Moment of torque

The moment of torque (τ) of an electric dipole in an external electric field measures the tendency of the dipole to align with the field.

When the dipole undergoes an electric field, it undergoes , the torque acts to orient the dipole so as to reduce the angle θ

$$\vec{\tau} = \vec{p} \times \vec{E}$$
$$|\vec{\tau}| = |\vec{E}||\vec{p}|\ \sin\theta$$

V chapter 4 : Gauss's theorem

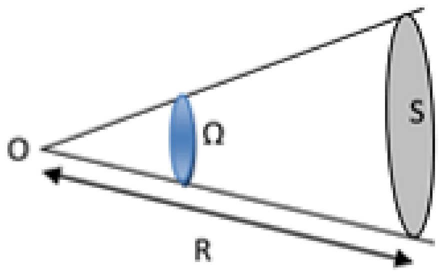
1. Introduction

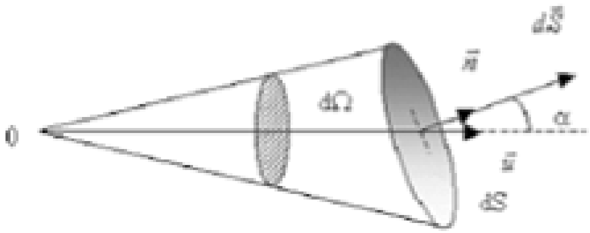
Gauss' theorem allows you to quickly calculate the electric field created by symmetrical charge distributions. First, we must define the notions of the Solid Angle and the flow of the electric field through a surface.

2. Solid angle concept

We saw in the previous study plane angles. But when it comes to spatial geometry, we find the solid angle. The solid angle is an “angle” in space, consider a sphere with center O and radius r

We define the solid angle Ω under which we see a surface (S), from a point O, contained in a cone with vertex O.





3. Concept of flow of the electric field through any surface

The flow of the field $\vec{E}(M)$ created at one point M by a charge distribution Q through a closed surface (S) is defined by:

$$\Phi_S = \oiint_S \vec{E}(M)\,\overrightarrow{ds}(M)$$

- With $\overrightarrow{ds}$ elementary surface vector: $\overrightarrow{ds} = ds.\vec{n}$ And $\vec{n}$ unit vector

4. General calculation method

- Find a closed surface passing through the point M where you want to calculate the field
- Write the definition of the flow

$$\Phi_S = \oiint_S \vec{E}.\overrightarrow{ds}$$

- Apply Gauss' theorem after calculating the algebraic charge inside the surface.

5. Example 1

Calculation of the electrostatic field created by a wire of infinite length and constant linear density λ positive by application of Gauss' theorem.

By application of Gauss's Theorem calculate the electrostatic field created by this distribution at a point located at distance x from the wire.

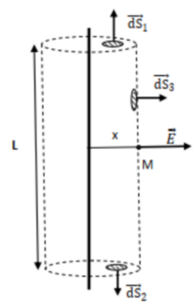
6. Solution

- Let us take as a closed surface (surface of *Gauss*), a cylinder of radius x and length l and axis the infinite wire.

- By reason of symmetry the field $\vec{E}$ is radial (carried by ox)

$\Phi(\vec{E}) = \Phi_{S_1}(\vec{E}) + \Phi_{S_2}(\vec{E}) + \Phi_{S_3}(\vec{E})$

$\Phi(\vec{E}) = \oiint_{S_1} \vec{E}.\overrightarrow{dS_1} + \oiint_{S_2} \vec{E}.\overrightarrow{dS_2} + \oiint_{S_3} \vec{E}.\overrightarrow{dS_3}$



- Field $\vec{E}$ is perpendicular to the normal at any point on both bases $\overrightarrow{S_1}$ And $\overrightarrow{S_2}$
- So : $\vec{E}.\overrightarrow{dS_1} = \vec{E}.\overrightarrow{dS_2} = 0 \Rightarrow \quad \Phi_{S_1}(\vec{E}) = \Phi_{S_2}(\vec{E}) = 0$
- Also, the field $\vec{E}$ is parallel to the normal of the lateral surface $\overrightarrow{S_3}$
- $\Phi_{S_3}(\vec{E}) = \oiint_{S_3} E.dS_3 = E.S_3 = E.2\pi.x.l$

2)The total charge contained in the surface of *Gauss* is .

$$Q = \int dq = \lambda \int dl = \lambda.l$$

By applying the theorem of *Gauss*

$$E.2\pi.x.l = \frac{\lambda l}{\varepsilon_0}$$

7. Example 2

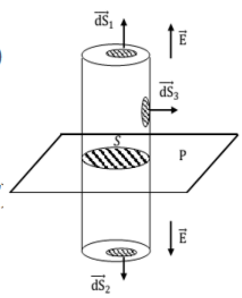
Calculation of the electrostatic field created at any point M in the space of an infinite plane(P) of uniform surface density σ positive by application of Gauss' theorem.

Calculate the electrostatic field generated by two infinite perpendicular planes, and with respective charge densities σ and 2σ.

8. Solution

- 1)-Let us take as the surface of *Gauss* a cylinder with an axis perpendicular to the plane. Because of symmetry the field $\vec{E}$ is perpendicular to the plane (P) The flow of the vector $\vec{E}$ emerging from the surface of *Gauss* is
- $\Phi(\vec{E}) = \Phi_{S_1}(\vec{E}) + \Phi_{S_2}(\vec{E}) + \Phi_{S_3}(\vec{E})$
- $\Phi(\vec{E}) = \oiint_{S_1} \vec{E}.\overrightarrow{dS_1} + \oiint_{S_2} \vec{E}.\overrightarrow{dS_2} + \oiint_{S_3} \vec{E}.\overrightarrow{dS_3}$

Field $\vec{E}$ is perpendicular to the normal of the lateral surface $\overrightarrow{S_3} \Rightarrow \Phi_{S_3} = 0$



- On the other hand, we have nothing to do with the two bases S_1And S_2

field $\vec{E}$ is parallel to the normal so

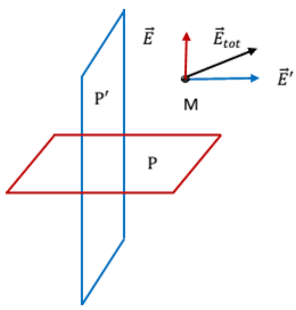
$$\Phi(\vec{E}) = \Phi_{S_1} + \Phi_{S_2} = ES_1 + ES_2 = 2ES_1 = 2ES \quad (S_1 = S_2 = S$$

- The charge contained in the surface of *Gauss* is
- $Q = \iint_s dq = \sigma \iint_s dS = \sigma S$ withe $(S_1 = S_2 = S)$
- We apply Gauss' theorem:
- $\Phi(\vec{E}) = 2ES = \frac{\sigma S}{\varepsilon_0} \Rightarrow E = \frac{\sigma}{2\varepsilon_0}$

9. B

- 2)-By analogy with question 1, the field $\vec{E'}$ created by the plan$(P')$is

$$E' = \frac{\sigma}{\varepsilon_0}$$



- Field $\vec{E}$ resulting is then:
- $\vec{E}_{tot} = \vec{E} + \vec{E'}$
- $E_{tot} = \sqrt{E^2 + E'^2}$

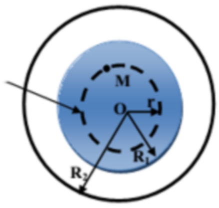
$$= \sqrt{(\frac{\sigma}{2\varepsilon_0})^2 + (\frac{\sigma}{\varepsilon_0})^2} = \frac{\sqrt{5}}{2}\frac{\sigma}{\varepsilon_0} \Rightarrow E_{tot} = \frac{\sqrt{5}}{2}\frac{\sigma}{\varepsilon_0}$$

10. Example 3

11. Solution

Given the symmetry of the problem, the field is radial.

- ***a) 1st case:*** $r < R_1$



- The flux Φ leaving the Gaussian sphere is:

$$\Phi = E_1(r)\,S_G = E_1(r)\,4\pi r^2 \;(S_G = S_{Gauss})$$

- The internal charge of the Gaussian sphere is:

$$\sum q_{int} = \int_0^r \rho dv = \int_0^r \rho 4\pi r^2 dr = \frac{4\rho\pi r^3}{3}$$

- *We will therefore have:*

$$\Phi = E_1\,S = \frac{\sum q_{int}}{\varepsilon_0} \Rightarrow E_1 4\pi r^2 = \frac{4\rho\pi r^3}{3\varepsilon_0} \Rightarrow E_1 = \frac{\rho r}{3\varepsilon_0}$$

12. B

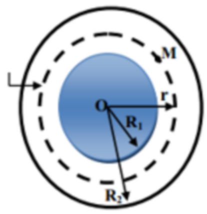
b) ***2nd case:*** $R_1 < r < R_2$

- $\Phi = E_2(r)\,S_G = E_2(r)\,4\pi r^2 \;(S_G = S_{Gauss})$

- The internal charge of the Gaussian sphere is:

- $\sum q_{int} = \int_0^{R_1} \rho dv = \int_0^{R_1} \rho 4\pi r^2 dr = \frac{4\rho\pi R_1^3}{3}$

He comes :



- $\Phi = E_2\,S = \frac{\sum q_{int}}{\varepsilon_0}$
- $E_2 = \frac{\rho R_1^3}{3\varepsilon_0 r^2}$
- $\Rightarrow E_2 4\pi r^2 = \frac{4\rho\pi R_1^3}{3\varepsilon_0}$

13. C

c) 3^{rd} case: $r > R_2$

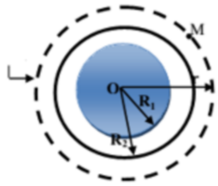
$$\Phi = E_3(r)\, S_G = E_3(r)\, 4\pi r^2 \;(S_G = S_{Gauss})$$

- The internal charge of the Gaussian sphere is:

$$\sum q_{int} = q_{R_1} + q_{R_2} = \frac{4\rho\pi R_1^3}{3} + \int_0^{R_2} \sigma dS = \frac{4\rho\pi R_1^3}{3} + \sigma \int_0^{R_2} 8\pi r dr = \frac{4\rho\pi R_1^3}{3} + 4\pi\sigma R_2^2$$

- Eventually :

$$E_3(r)\, 4\pi r^2 = \frac{4\rho\pi R_1^3}{3\varepsilon_0} + \frac{4\rho\pi R_2^2}{\varepsilon_0} \Rightarrow E_3(r) = \frac{\rho R_1^3 + 3\sigma R_2^2}{3\varepsilon_0 r^2}$$



VI Finale Test

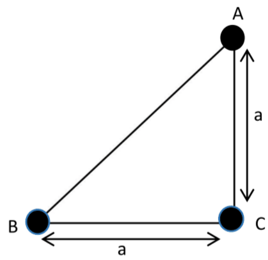
1. Final test

Course question

1.Define the electrostatic dipole. Represent the dipole moment (draw a diagram).

Exercise 01

Consider an isosceles right triangle ABC with equal sides (a) and three charges q_A = -q, q_B = +q and q_C = +q situated at points A, B and C respectively . We give : $q = 1\mu C.$ a = 1cm



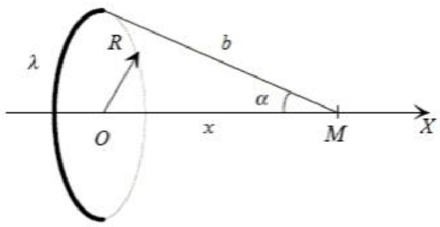
1. Represent graphically all the forces exercised by the three charges (without calculating them)
2. Calculate the electric field E and the potential V created by the charges on A
3. Deduce the force exercised on the charge situated at point A
4. Calculate the potential energy of q_A at point A

Exercise 02

A linear charge λ is distributed uniformly on a ring-shaped wire[3] of radius[4] R. (figure opposite).

1-Calculate the electric field produced by the wire[5] at point M located on the axis OX at a distance x from center W.

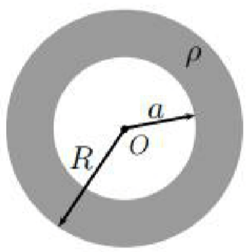
2- Calculate the electric potential created by ring-shaped wire of radius R



Exercise 03

Consider a spherical cavity[6] of radius $\boldsymbol{a}$ at the center of a sphere non-conductive, center O and radius A. The rest of the sphere has a volume density of positive charge and uniform ρ

1. Determine the electric field created at any point $M(r)$ in space: $r < a, a < r < R \, and \, r > R.$
2. Deduce the electrostatic potential at any point $M(r)$ in space: $r < a, a < r < R \, and \, r > R$, knowing that $V\,(r \to \infty) = 0.$



the vertices[1] رؤوس / diamond[2] المعين/ ring-shaped wire[3] خيط على شكل حلقة/ radius[4] نصف القطر /wire[5] خيط/ spherical cavity[6] كرة مجوفة

Conclusion

Congratulations on completing this course on electrostatics! Throughout the journey, you've developed a deep understanding of electric charges, fields, and potentials, exploring key concepts such as point charges, continuous charge distributions, electric dipoles, and Gauss's Theorem.

By mastering these topics, you've equipped yourself with the tools to analyze and solve complex electrostatic problems, both theoretically and practically. You've also gained insight into how these principles apply to real-world scenarios, laying a strong foundation for future studies in physics and engineering.

As you move forward, remember that the concepts learned here are fundamental to many areas of science and technology. Whether in advanced physics courses or in professional applications, the knowledge you've gained will be invaluable. Keep exploring, questioning, and applying these principles as you continue your academic and professional journey.

Thank you for your dedication, and best of luck in all your future endeavors!