**Badji- Mokhtar University -ANNABA Faculty of technology**

**Computer science department & Electronics**

**Department 1st year Computer sciences&**

**automatics (2023-2024) Online courses**

**Coursework Exercise 3 of Physics 2**

**Gauss's theorem**

**Exercise 1:**

- Let us take as a closed surface (surface of *Gauss*), a cylinder of radius $x$ and length $l$ and axis the infinite wire.

By reason of symmetry the field $\vec{E}$ is radial (carried by $ox$)

$\Phi(\vec{E}) = \Phi_{S_1}(\vec{E}) + \Phi_{S_2}(\vec{E}) + \Phi_{S_3}(\vec{E})$

$\Phi(\vec{E}) = \oiint_{S_1} \vec{E}.\overrightarrow{dS_1} + \oiint_{S_2} \vec{E}.\overrightarrow{dS_2} + \oiint_{S_3} \vec{E}.\overrightarrow{dS_3}$

Field $\vec{E}$ is perpendicular to the normal at any point on both bases $\overrightarrow{S_1}$ And $\overrightarrow{S_2}$

So: $\vec{E}.\overrightarrow{dS_1} = \vec{E}.\overrightarrow{dS_2} = 0 \Rightarrow \ \Phi_{S_1}(\vec{E}) = \Phi_{S_2}(\vec{E}) = 0$

Also, the field $\vec{E}$ is parallel to the normal of the lateral surface $\overrightarrow{S_3}$

$\Phi_{S_3}(\vec{E}) = \oiint_{S_3} E.dS_3 = E.S_3 = E.2\pi.x.l$

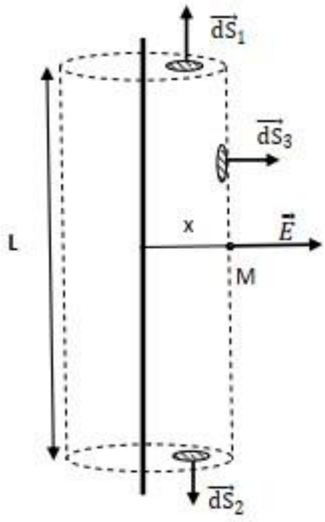
The total charge contained in the surface of *Gauss* is.

$Q = \int dq = \lambda \int dl = \lambda.l$

By applying the theorem of *Gauss*

$E.2\pi.x.l = \frac{\lambda l}{\varepsilon_0}$

$$\boxed{\boldsymbol{E = \frac{\lambda}{2\pi x\varepsilon_0}}}$$




$$\mathbf{V} = \int -\boldsymbol{E}d\boldsymbol{x} = -\int \frac{\boldsymbol{\lambda}}{\boldsymbol{2\pi x\varepsilon_0}} d\boldsymbol{x} = -\frac{\boldsymbol{\lambda}}{\boldsymbol{2\pi\varepsilon_0}} \ln(\boldsymbol{r}) + \boldsymbol{cte}$$

**Exercise 2:**

Let us take as the surface of *Gauss* a cylinder with an axis perpendicular to the plane. Because of symmetry the field $\vec{E}$ is perpendicular to the plane $(P)$ The flow of the vector $\vec{E}$ emerging from the surface of *Gauss is*:

$\Phi(\vec{E}) = \Phi_{S_1}(\vec{E}) + \Phi_{S_2}(\vec{E}) + \Phi_{S_3}(\vec{E})$

$\Phi(\vec{E}) = \oiint_{S_1} \vec{E}.\overrightarrow{dS_1} + \oiint_{S_2} \vec{E}.\overrightarrow{dS_2} + \oiint_{S_3} \vec{E}.\overrightarrow{dS_3}$

Field $\vec{E}$ is perpendicular to the normal of the lateral surface $\overrightarrow{S_3} \Rightarrow \Phi_{S_3} = 0$

On the other hand, we have nothing to do with the two bases $S_1$And $S_2$ ,

field $\vec{E}$ is parallel to the normal so

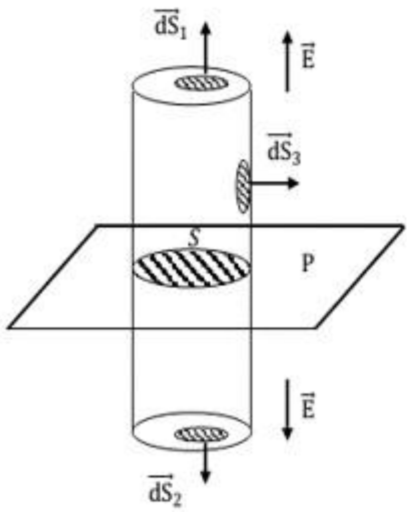
$\Phi(\vec{E}) = \Phi_{S_1} + \Phi_{S_2} = ES_1 + ES_2 = 2ES_1 = 2ES \quad (S_1 = S_2 = S)$

So the charge contained in the surface of *Gauss* is

$Q = \iint_S dq = \sigma \iint_S dS = \sigma S$ withe $(S_1 = S_2 = S)$

We apply Gauss' theorem:

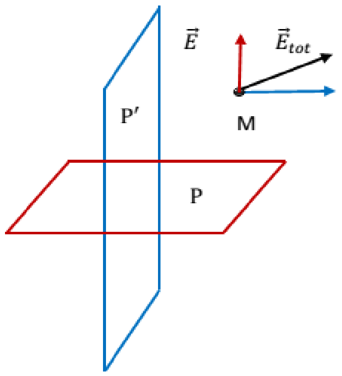
$\Phi(\vec{E}) = 2ES = \frac{\sigma S}{\varepsilon_0} \Rightarrow E = \frac{\sigma}{2\varepsilon_0}$



2) By analogy with question 1, the field $\vec{E'}$ created by the plan$(P')$is

$$E' = \frac{\sigma}{\varepsilon_0}$$

Total Field $\vec{E}$ resulting is then:




$$\vec{E}_{tot} = \vec{E} + \vec{E}'$$

$$E_{tot} = \sqrt{E^2 + E'^2}$$

$$E_{tot} = \sqrt{(\frac{\sigma}{2\varepsilon_0})^2 + (\frac{\sigma}{\varepsilon_0})^2} = \frac{\sqrt{5}}{2}\frac{\sigma}{\varepsilon_0}$$

$$\boxed{\boldsymbol{E_{tot} = \frac{\sqrt{5}}{2}\frac{\sigma}{\varepsilon_0}}}$$

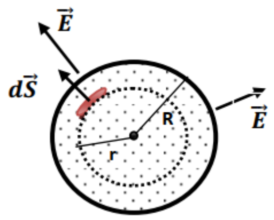
**Exercise 3:**

By application of Gauss's Theorem, let us calculate the electrostatic field created by a sphere with center O and radius R charged with a constant volume density positive $\rho$

For reasons of symmetry, the vector $E$ is radial and has the same modulus at any point of a sphere with center O and radius r (Gaussian surface)

**1st case r < R:**

$$\Phi(\vec{E}) = \oiint_S \vec{E_1}.\vec{dS} = \oiint_S E_1 \,.dS = E_1.S = E_1 4\pi r^2$$




The total charge of the sphere is

$$Q = \iiint \rho dV = \rho. \int_0^r dV = \rho.V$$

$$Q = \rho\frac{4}{3}\pi r^3$$

By applying gauss's theorem

$$E_1 4\pi r^2 = \frac{\rho 4\pi r^3}{3\varepsilon_0}$$

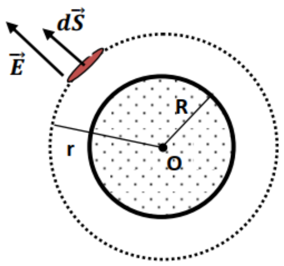
$$\boxed{\boldsymbol{E_1 = \frac{\rho}{3\varepsilon_0} r}}$$

**2nd case r > R:**

$$\Phi(\vec{E}) = \oiint_S \vec{E_2}.\vec{dS} = \oiint_S E_2 \,.dS = E_2.S = E_2 4\pi r^2$$

The total charge of the sphere is

$$Q = \iiint \rho dV = \rho. \int_0^R dV = \rho.V$$




$$Q = \rho\frac{4}{3}\pi R^3$$

By applying gauss's theorem

$$E_2 4\pi r^2 = \frac{\rho 4\pi R^3}{3\varepsilon_0}$$

$$\boxed{\boldsymbol{E_2 = \frac{\rho R^3}{3\varepsilon_0 r^2}}}$$

**2) Electrical potential**

r < R: $V_1 = -\frac{\rho r^2}{6\varepsilon_0} + C_1$

r >R: $V_2 = \frac{\rho R^3}{6\varepsilon_0 r} + C_2$

By writing the boundary conditions we have

$$V_2\,(\infty) = 0 \Rightarrow C_2 = 0$$

$$V_1\,(R) = V_2\,(R) \Rightarrow C_1 = \frac{\rho R^2}{2\varepsilon_0}$$