Exercice 01: Solution (Serie 01)

We have: $e = 1{,}6 \cdot 10^{-19}\,C$ / $m_e = 9{,}1\,10^{-31}\,Kg$

$g = 9{,}81\ m.s^{-2}$ / $m_p = 1{,}672\,10^{-27}\,Kg$

$G = 6{,}67.10^{-11}\,N.m^2.Kg^{-2}$ / $K = 9.10^{9}\,N.m^2.C^{-2}$

and $a = 0{,}53 \cdot 10^{-10}\,m$.

1/ The electrostatic force:

$$\vec{F_e} = \frac{K q_1 q_2}{r^2}\vec{U_r}$$

$$\text{So} \Rightarrow F_e = \frac{K|q_1 q_2|}{r^2} = \frac{K|q_e q_p|}{a^2}$$

$$AN: F_e = \frac{9\times10^9 \times 1{,}6\times10^{-19}\times 1{,}6\,10^{-19}}{(0{,}53.10^{-10})^2}$$

$$\boxed{F_e \simeq 8{,}2\,10^{-8}\,N}$$

2/ Universal force of attraction

$$\vec{F_g} = -G\frac{m_1 m_2}{r^2}\vec{U_r}$$

$$\text{So} \Rightarrow F_g = G\frac{m_1 m_2}{r^2} = G\frac{m_e m_p}{a^2}$$

$$F_g = \frac{6.67\,10^{-11}\times 9{,}1.10^{-31}\times 1{,}672\times10^{-27}}{(0{,}53.10^{-10})^2}$$

$$\boxed{F_g \simeq 3{,}6.10^{-46}\,N}$$

Comparison:

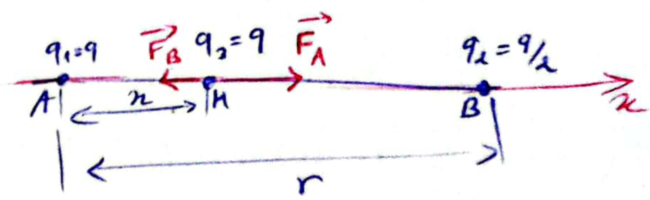
We conclude that gravitational forces are negligible compared with electrostatic forces in the case of small particles (electrons, protons, ions ---).

Exercice 02:

• Given that the three charges have the same sign, we therefore witness the phenomenon of repulsion between the charges

1. The charge q_3 is therefore subject to the resultant $\vec{F_M}$ of the two forces applied by q_1 and q_2:

$$\vec{F_M} = \vec{F_A} + \vec{F_B}$$



Proj on the x axis, we will have:

$$F_M = F_A - F_B = \frac{K q_1 q_3}{x^2} - \frac{K q_2 q_3}{(r-x)^2}$$

$$F_M = \frac{Kq^2}{x^2} - \frac{Kq^2}{2(r-x)^2}$$

$$\text{So} \Rightarrow F_M = Kq^2\left(\frac{1}{x^2} - \frac{1}{2(r-x)^2}\right)$$

2/ The charge q_3 is in equilibrium position when:

$$\sum \vec{F} = 0 \Rightarrow F_M = F_1 - F_B = 0$$

$\Rightarrow$ So $Kq^2\left(\frac{1}{x^2} - \frac{1}{2(r-x)^2}\right) = 0$

$$\Rightarrow 2(r-x)^2 - x^2 = 0$$

$$\Rightarrow 2(3-x)^2 - x^2 = 2x^2 - 12x + 18 - x^2 = 0$$

$$\Rightarrow \boxed{x^2 - 12x + 18 = 0}$$

we have a second degree trinomial, with a discriminant ($\Delta > 0$)

$$\Delta = b^2 - 4ac = (-12)^2 - (4\times1\times18) = 72 > 0$$

$\Rightarrow$ So two roots:

$$x_1 = \frac{-b-\sqrt{\Delta}}{2a} = \frac{12-\sqrt{72}}{2} \approx 1,75\ \text{cm}$$

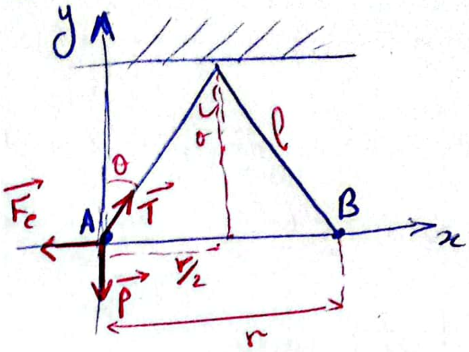
$$x_2 = \frac{-b+\sqrt{\Delta}}{2a} = \frac{12+\sqrt{72}}{2} \approx 10,24\ \text{cm}$$

So $\boxed{x_0 \approx 1,75\ \text{cm}}$ is the abscissa for which the charge q_3 is in equilibrium position.

As for the other root $\boxed{x_2 \approx 10,24\ \text{cm}}$ it is excluded since $x_1 > x$.

Exercice 03:

The charge q placed at point A is subject to:



- The electrostatic force $\vec{F}$ (the charge placed at B) exerts on the charge placed at A repulsive force "the two charges have the same sign"
- The tension of the wire $\vec{T}$
- The weight $\vec{P}$ ~~electrostatic equilibrium~~

At electrostatic equilibrium

$$\sum \vec{F}_{ext} = \vec{0} = \vec{F} + \vec{T} + \vec{P} = \vec{0}$$

axis projection:

$$\begin{cases} Ox: -F_e + T\sin\theta = 0 \\ Oy: -P + T\cos\theta = 0 \end{cases} \Rightarrow \begin{cases} F_e = \sin\theta = \frac{Kq^2}{r^2} \\ mg = T\cos\theta \end{cases}$$

$$\Rightarrow \frac{Kq^2}{r^2} = T\sin\theta = mg\frac{\sin\theta}{\cos\theta} = mg\tan\theta \cong mg\sin\theta$$

and we have: $\sin\theta = \frac{r/2}{l} \Rightarrow \frac{Kq^2}{r^2} = mg\frac{r}{2l}$

$$\Rightarrow r^3 = \frac{2Klq^2}{mg} \Rightarrow r = \left(\frac{2Klq^2}{mg}\right)^{1/3}$$

$$r = \left(\frac{2\times9\times10^{9}\times120\times10^{-2}\times(2,4\times10^{-8})^2}{10\times10^{-3}\cdot10}\right)^{1/3} \Rightarrow \boxed{r \approx 5\ \text{cm}}$$